1. $3x^2 - 13x + 4 = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ①

A B C

SNAPSHOT

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{13 \pm \sqrt{169-48}}{6} = \frac{13 \pm \sqrt{121}}{6} = \frac{13 \pm 11}{6}$$

$$= \frac{13+11}{6} \quad \frac{13-11}{6}$$

$$= \frac{24}{6} \quad \frac{2}{6} \quad \boxed{x = 4 \text{ and } 1/3}$$

$$3x^2 - 7x + 4 = 0$$

$$x = \frac{7 \pm \sqrt{49-48}}{2(3)} \qquad x = \frac{7 \pm \sqrt{1}}{6}$$

$$x = \frac{7+1}{6} \qquad x = \frac{8}{6} \qquad x = \frac{4}{3}$$

$$x = \frac{7-1}{6} \qquad x = \frac{6}{6} \qquad x = 1$$

Classwork:

Simplify: $\sqrt{41+24\sqrt{2}}$ in the form $a+b\sqrt{2}$

Solution: $(a+b\sqrt{2})^2 = \left(\sqrt{41+24\sqrt{2}}\right)^2$

$$a^2 + 2(a)(b\sqrt{2}) + (b\sqrt{2})^2 = 41 + 24\sqrt{2}$$

$$a^2 + 2ab\sqrt{2} + b^2 \cdot 2 = 41 + 24\sqrt{2}$$

$$a^2 + \underline{2ab\sqrt{2}} + 2b^2 = 41 + \underline{24\sqrt{2}}$$

$$\underline{a^2 + 2b^2} + \underline{2ab\sqrt{2}} = \underline{41} + \underline{24\sqrt{2}}$$

* set up a system of equations

i) $a^2 + 2b^2 = 41$ ii) $2ab\sqrt{2} = 24\sqrt{2}$

* solve for a in terms of b.

wisest: $\frac{2ab\sqrt{2}}{2b\sqrt{2}} = \frac{24\sqrt{2}}{2b\sqrt{2}}$ ← to get a by itself.

$\boxed{a = \frac{12}{b}}$ ← substitute into equation i

$$a^2 + 2b^2 = 41$$

$$\left(\frac{12}{b}\right)^2 + 2b^2 = 41 \Rightarrow \frac{(12)^2}{(b)^2} + 2b^2 = 41$$

$$b^2\left(\frac{144}{b^2} + 2b^2 = 41\right)$$ ← get rid of fraction so I multiply whole eq. by the denominator.

$$\frac{144\cancel{b^2}}{\cancel{b^2}} + 2b^2(b^2) = 41(b^2)$$

$$144 + 2b^4 = 41b^2$$ ← set equation equal to zero

$$-41b^2 \quad -41b^2$$

$$144 + 2b^4 - 41b^2 = 0$$ ← rearrange

$$2b^4 - 41b^2 + 144 = 0$$ ← use substitution $x = b^2$

$$2x^2 - 41x + 144 = 0$$ ← use quadratic

$$x = 16 \qquad x = \frac{9}{2}$$

Recall: $x = b^2$

$$x = 16 \qquad x = \frac{9}{2}$$

$$\sqrt{b^2} = \sqrt{16} \qquad \sqrt{b^2} = \sqrt{\frac{9}{2}}$$

$$\boxed{b = \pm 4} \qquad \boxed{b = \pm \frac{3}{\sqrt{2}}}$$

* go back and solve for a ...

$$a = \frac{12}{b}, \quad b = \pm 4 \;\&\; b = \pm \frac{3}{\sqrt{2}}$$

Solution 1	Sol. 2	Sol 3	Sol 4
$b = +4$	$b = -4$	$b = +\frac{3}{\sqrt{2}}$	$b = -\frac{3}{\sqrt{2}}$
$a = \frac{12}{b}$	$a = \frac{12}{b}$	$a = \frac{12}{b}$	$a = \frac{12}{b}$
$a = \frac{12}{4}$	$a = \frac{12}{-4}$	$a = \frac{12}{\left(\frac{3}{\sqrt{2}}\right)}$	$a = \frac{12}{b}$
$a = 3$	$a = -3$	$a = \frac{\left(\frac{12}{1}\right)}{\left(\frac{3}{\sqrt{2}}\right)}$	$a = \frac{12}{-\frac{3}{\sqrt{2}}}$
$\boxed{a=3,\ b=4}$	$\boxed{a=-3,\ b=-4}$	$a = \frac{12}{1} \cdot \frac{\sqrt{2}}{3}$	$a = \frac{\left(\frac{12}{1}\right)}{\left(-\frac{3}{\sqrt{2}}\right)}$
		$a = \frac{12\sqrt{2}}{3}$	$a = \frac{12}{1} \cdot -\frac{\sqrt{2}}{3}$
		$a = 4\sqrt{2}$	$a = -\frac{12\sqrt{2}}{3}$
		$\boxed{a=4\sqrt{2},\ b=\frac{3}{\sqrt{2}}}$	$a = -4\sqrt{2}$
			$\boxed{a=-4\sqrt{2},\ b=-\frac{3}{\sqrt{2}}}$

<table>
<tr><th>S1 (Solution)</th><th>S2</th><th>S3</th><th>S4</th></tr>
<tr>
<td>$a=3,\ b=4$
$\sqrt{41+24\sqrt{2}}=a+b\sqrt{2}$
$\left(\sqrt{41+24\sqrt{2}}\right)^2=\left(3+4\sqrt{2}\right)^2$
$41+24\sqrt{2}=9+2(3)(4\sqrt{2})+16\cdot 2$
$=9+24\sqrt{2}+32$
$=41+24\sqrt{2}$
$41+24\sqrt{2}=41+24\sqrt{2}$
✓</td>
<td>$a=-3,\ b=-4$
$\sqrt{41+24\sqrt{2}}=a+b\sqrt{2}$
$41+24\sqrt{2}=\underbrace{-3-4\sqrt{2}}_{\text{negative}}$
Does not work, need to be positive.
✗</td>
<td>$a=4\sqrt{2},\ b=\frac{3}{\sqrt{2}}$
$\sqrt{41+24\sqrt{2}}=a+b\sqrt{2}$
$\sqrt{41+24\sqrt{2}}=4\sqrt{2}+\frac{3}{\cancel{\sqrt{2}}}(\cancel{\sqrt{2}})$
$\sqrt{41+24\sqrt{2}}=4\sqrt{2}+3$
Same as the 1st solution
✓</td>
<td>$a=-4\sqrt{2},\ b=-\frac{3}{\sqrt{2}}$
$\sqrt{41+24\sqrt{2}}=-4\sqrt{2}+\frac{-3}{\sqrt{2}}\sqrt{2}$
$=-4\sqrt{2}-3$
Same as solution #2 doesn't work, is a negative number.</td>
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Classwork:

Simplify: $\sqrt{13-4\sqrt{3}}=a+b\sqrt{3}$

Find an a and b that make this true.

$$\left(\sqrt{13-4\sqrt{3}}\right)^2=\left(a+b\sqrt{3}\right)^2$$

$$13-4\sqrt{3}=a^2+2ab\sqrt{3}+b^2\cdot 3$$

$$13-4\sqrt{3}=a^2+2ab\sqrt{3}+3b^2$$

{SIDE NOTES}

$$\sqrt{2}^{\,2} \Rightarrow 2$$

$$\sqrt{2} = \left(2^{1/2}\right)^2 \Rightarrow 2^{2/2} = 2^1$$
$$= 2$$

Why $\sqrt{2}^{\,2} \Rightarrow 2$

$$(\overset{1}{a} + \overset{2}{b\sqrt{3}})^2 = (a+b\sqrt{3})(a+b\sqrt{3})$$

$$\underbrace{a^2}_{\text{1st term squared}} + \underbrace{2(a)(b\sqrt{3})}_{\substack{\text{Product of}\\ \text{the first and}\\ \text{second terms}\\ \text{doubled.}}} + \underbrace{b^2(\sqrt{3}^{\,2})}_{\substack{\text{last term}\\ \text{squared}}}$$

How Mrs. Vallejos does $(a+b\sqrt{3})^2$ quickly.