1979 AB 1

Given the function f defined by $f(x) = 2x^3 - 3x^2 - 12x + 20$.

a. Find the zeros of f.

b. Write an equation of the line normal to the graph of f at $x = 0$.

a. Find the x- and y-coordinates of all points on the graph of f where the line tangent to the graph is parallel to the x-axis.

1979 AB 2

A function f is defined by $f(x) = x\, e^{-2x}$ with domain $0 \le x \le 10$.

a. Find all values of x for which the graph of f is increasing and all values of x for which the graph decreasing.

b. Give the x- and y-coordinates of all absolute maximum and minimum points on the graph of f. Justify your answers.

1979 AB 3 BC 3

Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.

1979 AB 4 BC 1

A particle moves along a line so that at any time t its position is given by

$x(t) = 2\pi t + \cos 2\pi t$.

a. Find the velocity at time t.

b. Find the acceleration at time t.

c. What are all values of t, $0 \le t \le 3$, for which the particle is at rest?

d. What is the maximum velocity?

1979 AB 5 BC 5

Let R be the region bounded by the graph of $y = (1/x)\ln(x)$, the x-axis, and the line $x = e$.

a. Find the area of the region R.

b. Find the volume of the solid formed by revolving the region R about the y-axis.

1979 AB 6

Given the function f where $f(x) = x^2 - 2x$ for all real numbers x.

a. On the axes provided, sketch the graph of $y = |f(x)|$.

b. Determine whether the derivative of $|f(x)|$ exists at $x = 0$. Justify your answer.

c. On the axes provided, sketch the graph of $y = f(|x|)$.

d. Determine whether $f(|x|)$ is continuous at $x = 0$. Justify your answer.

1979 AB 7

Let f be the function defined by f(x) $x^3 + ax^2 + bx + c$ and having the following properties:

(i) The graph of f has a point of inflection at (0,-2).

(ii) The average (mean) value of f(x) on the closed interval [0,2] is -3.

a. Determine the values of a, b, and c.

b. Determine the value of x that satisfies the conclusion of the Mean Value Theorem for f on the closed interval [0,3].

1980 AB 1

Let R be the region enclosed by the graphs of $y = x^3$ and $y = \sqrt{x}$.

a. Find the area of R.

b. Find the volume of the solid generated by revolving R about the x-axis.

1980 AB 2

A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ and the x-axis.

a. Find the x- and y-coordinates of C so that the area of the rectangle ABCD is a maximum.

b. The point C moves along the curve with its x-coordinate increasing at the constant rate of 2 units per second. Find the rate of change of the area of rectangle ABCD when $x = \frac{1}{2}$.

1980 AB 3

Let $f(x) = \ln(x^2)$ for $x > 0$ and $g(x) = e^{2x}$ for $x \geq 0$. Let H be the composition of f with g, that is H(x) = f(g(x)), and let K be the composition of g with f, that is, K(x) = g(f(x)).

a. Find the domain of H and write an expression for H(x) that does not contain the exponential function.

b. Find the domain of K and write an expression for K(x) that does not contain the exponential function.

c. Find an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f, and find the domain of f^{-1}.

1980 AB 4 BC 1

The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

a. Write an expression for the velocity v, in terms of time t, if v = 5 when t = 0.

b. During the time when the velocity increases from 5 to 15, how far does the particle travel?

c. Write an expression for the position s, in terms of time t, of the particle if $s = 0$ when $t = 0$.

1980 AB 5 BC 2

Given the function f defined by $f(x) = \cos(x) - \cos^2 x$ for $-\pi \le x \le \pi$.

a. Find the x-intercepts of the graph of f.
b. Find the x- and y-coordinates of all relative maximum points of f. Justify your answer.
c. Find the intervals on which the graph of f is increasing.
d. Using the information found in parts a, b, and c, sketch the graph of f on the axes provided.

1980 AB 6 BC 4

Let $y = f(x)$ be the continuous function that satisfies the equation $x^4 - 5x^2y^2 + 4y^4 = 0$ and whose graph contains the points (2,1) and (-2,-2). Let $\underline{\ell}$ be the line tangent to the graph of f at $x = 2$.

a. Find an expression for y'.
b. Write an equation for line $\underline{\ell}$.

c. Give the coordinates of a point that is on the graph of f but is not on line $\underline{\ell}$.
d. Give the coordinates of a point that is on line $\underline{\ell}$ but is not on the graph of f.

1980 AB 7

Let p and q be real numbers and let f be the function defined by:

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2, & \text{for } x \le 1 \\ qx + p, & \text{for } x > 1 \end{cases}$$

a. Find the value of q, in terms of p, for which f is continuous at $x = 1$.
b. Find the values of p and q for which f is continuous at $x = 1$.
c. If p and q have the values determined in part b, is f'' a continuous function? Justify your answer.

1981 AB 1

Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at the point where $x = 1$.
c. Find the x-coordinate of each point at which the line tangent to the

graph of f is parallel to the line y = -2x + 4.

1981 AB 2

Let R be the region in the first quadrant enclosed by the graphs of $y = 4 - x^2$, $y = 3x$, and the y-axis.

a. Find the area of the region R.
b. Find the volume of the solid formed by revolving the region R about the x-axis.

1981 AB 3 BC 1

Let f be the function defined by $f(x) = 12x^{(2/3)} - 4x$.

a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

1981 AB 4

Let f be the function defined by $f(x) = 5^{\sqrt{2x^2-1}}$.

a. Is f an even or odd function? Justify your answer.
b. Find the domain of f.
c. Find the range of f.
d. Find $f'(x)$.

1981 AB 5 BC 2

Let f be a function defined by $f(x) = \begin{cases} 2x+1, \text{ for } x \le 2 \\ \frac{1}{2}x^2 + k, \text{ for } x > 2 \end{cases}$.

a. For what values of k will f be continuous at x = 2? Justify your answer.
b. Using the value of k found in part a, determine whether f is differentiable at x = 2. Use the definition of the derivative to justify your answer.
c. Let k = 4. Determine whether f is differentiable at x = 4. Justify your answer.

1981 AB 6 BC 4

A particle moves along the x-axis so that at time t its position is given by

$x(t) = \sin(\pi t^2)$ for $-1 \leq t \leq 1$.

a. Find the velocity at time t.
b. Find the acceleration at time t.
c. For what values of t does the particle change direction?
d. Find all values of t for which the particle is moving to the left.

1981 AB7

Let f be a continuous function that is defined for all real numbers x and that has the following properties.

(i) $\int_1^3 f(x) = \frac{5}{2}$

(ii) $\int_1^5 f(x) = 10$

a. Find the average (mean) value of f over the closed interval [1,3].
b. Find the value of $\int_3^5 (2f(x)+6)dx$
c. Given that f(x) = ax + b, find the values of a and b.

1982 AB 1

A particle moves along the x-axis in such a way that its acceleration at time t for t > 0 is given by a(t) = $\frac{3}{t^2}$. When t = 1, the position of the particle is 6 and the velocity is 2.

a. Write an equation for the velocity, v(t), of the particle for all t > 0.
b. Write an equation for the position, x(t), of the particle for all t > 0.
c. Find the position of the particle when t = e.

1982 AB 2

Given that f is the function defined by f(x) = $\frac{x^3 - x}{x^3 - 4x}$.

a. Find the $\lim_{x\to 0} f(x)$.
b. Find the zeros of f.
c. Write an equation for each vertical and each horizontal asymptote to the graph of f.

d. Describe the symmetry of the graph of f.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

1982 AB 3 BC 1

Let R be the region in the <u>first quadrant</u> that is enclosed by the graph of y = tan(x), the x-axis, and the line x = $\frac{\pi}{3}$.

a. Find the area of R.
b. Find the volume of the solid formed by revolving R about the x-axis.

1982 AB 4

A ladder 15 feet long is leaning against a building so that the end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.

1982 AB 5 BC 2

Let f be the function defined by $f(x) = (x^2 + 1)e^{-x}$ for $-4 \le x \le 4$.

a. For what value of x does f reach its absolute maximum? Justify your answer.
b. Find the x-coordinates of all points of inflection of f. Justify your answer.

1982 AB 6 BC 3

A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank?

1982 AB 7

For all real numbers x, f is a differentiable function such that f(-x) = f(x). Let f(p) = 1 and $f'(p) = 5$ for some p > 0.

a. Find $f'(p)$.
b. Find $f'(0)$.
c. If l_1 and l_2 are lines tangent to the graph of f at (-p,1) and (p,1), respectively, and if l_1 and l_2 intersect at point Q, find the x- and y-coordinates of Q in terms of p.

1983 AB 1

Let f be the function defined by $f(x) = -2 + \ln(x^2)$.

a. For what real numbers x is f defined?
b. Find the zeros of f.
c. Write an equation for the line tangent to the graph of f at $x = 1$.

1983 AB 2

A particle moves along the x-axis so that at time t its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$.

a. What is the velocity of the particle at $t = 0$?
b. During what time intervals is the particle moving to the left?
c. What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

1983 AB 3 BC 1

Let f be the function defined for $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$ by $f(x) = x + \sin^2 x$.

a. Find all values of x for which $f'(x) = 1$.
b. Find the x-coordinates of <u>all</u> minimum points of f. Justify your answer.
c. Find the x-coordinates of <u>all</u> inflection points of f. Justify your answer.

1983 AB 4

Let R be the shaded region between the graph of $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$ and the x-axis from $x = 0$ to $x = 1$.

a. Find the area of R by setting up and integrating a definite integral.
b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the x-axis.
c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the line $x = 1$.

1983 AB 5 BC 3

At time $t = 0$, a jogger is running at a velocity of 300 meters per minute. The jogger is slowing down with a negative acceleration that is directly proportional to time t. This brings the jogger to a stop in10 minutes.

a. Write an expression for the velocity of the jogger at time t.
b. What is the total distance traveled by the jogger in that 10-minute interval?

1983 AB 6

Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis.

a. Find the area of the region R.
b. Find the volume of the solid generated when R is revolved about the x-axis.
c. The vertical line x = k divides the region R into two regions such that when these two regions are revolved about the x-axis, the generate solids with equal volumes. Find the value of k.

1984 AB 1

A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by a(t) = 6 t + 6. At time t = 0, the velocity of the particle is -9, and its position is -27.

a. Find v(t), the velocity of the particle at any time $t \geq 0$.
b. For what values of $t \geq 0$ is the particle moving to the right?
c. Find x(t), the position of the particle at any time $t \geq 0$.

1984 AB 2

Let f be the function defined by $f(x) = \frac{x + \sin x}{\cos x}$ for $\frac{-\pi}{2} < x < \frac{\pi}{2}$.

a. State whether f is an even function or an odd function. Justify your answer.
b. Find $f'(x)$.
c. Write an equation of the line tangent to the graph of f at the point where x = 0.

1984 AB 3 BC 1

Let R be the region enclosed by the x-axis, the y-axis, the line x = 2, and the $y = 2e^x + 3x$.

a. Find the area of R by setting up and evaluating the definite integral. Your work must include an antiderivative.
b. Find the volume of the solid generated by revolving R about the <u>y-axis</u> by setting up and evaluating a definite integral. Your work must include an antiderivative.

1984 AB 4 BC 3

A function f is continuous on the closed interval [-3,3] such that f(-3) = 4 and f(3) = 1. The functions $f'(x)$ and $f''(x)$ have the properties given in the table below.

x	$f'(x)$	$f''(x)$
$-3<x<-1$	Positive	Positive
$x=-1$	Fails to exist	Fails to exist
$-1<x<1$	Negative	Positive
$x=1$	Zero	Zero
$1<x<3$	Negative	negative

a. What are the x-coordinates of all absolute maximum and minimum points of f on the interval [-3,3]? Justify your answer.
b. What are the x-coordinates of all points of inflection on the interval [-3,3]? Justify your answer.
c. On the axes provided, sketch a graph that satisfies the given properties of f.

1984 AB 5

The volume V of a cone is increasing at the rate of 28 π cubic inches per second. At the instant when the radius r on the cone is 3 units, its volume is 12 π cubic units and the radius is increasing at $\frac{1}{2}$ unit per second.

a. At the instant when the radius of the cone is 3 units, what is the rate of change of the area of its base?
b. At the instant when the radius of the cone is 3 units, what is the rate of change of its height h?
c. At the instant when the radius of the cone is 3 units, what is the instantaneous rate of change of the area of its base with respect to its height h?

1985 AB 1

Let f be the function given by $f(x)=\dfrac{2x-5}{x^2-4}$.

a. Find the domain of f.
b. Write an equation for each vertical and each horizontal asymptote for the graph of f.
c. Find $f'(x)$.
d. Write an equation for the line tangent to the graph of f at the point (0,f(0)).

1985 AB 2 BC 1

A particle moves along the x-axis with acceleration given by $a(t) = \cos(t)$ for $t \geq 0$. At $t = 0$ the velocity $v(t)$ of the particle is 2 and the position $x(t)$ is 5.

a. Write an expression for the velocity $v(t)$ of the particle.
b. Write an expression for the position $x(t)$.
c. For what values of t is the particle moving to the right? Justify your answer.
d. Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

1985 AB 3

Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^{x}$ and $x = \ln 4$.

a. Find the area of R by setting up and evaluating a definite integral.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the x-axis.
c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the y-axis.

1985 AB 4 BC 3

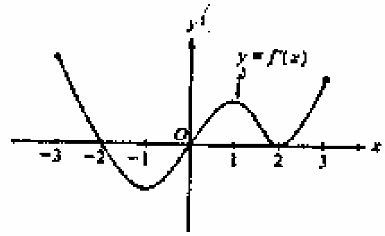
Let $f(x) = (14\pi)x^2$ and $g(x) = k^2 \sin \frac{\pi x}{2k}$ for $k > 0$.

a. Find the average value of f on [1,4].
b. For what value of k will the average value of g on [0,k] be equal to the average value of f on [1,4]?

1985 AB 5 BC 2

A balloon in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.

a. At this instant, what is the height of the cylinder?
b. At this instant, how fast is the height of the cylinder increasing?



1985 AB 6
The figure above shows the graph of f′, the derivative of a function f. The domain of the function f is the set of all x such that $-3 \le x \le 3$.

a. For what values of x, $-3 < x < 3$, does f have a relative maximum? A relative minimum? Justify your answer.
b. For what values of x is the graph of f concave up? Justify your answer.
c. Use the information found in parts (a) and (b) and the fact that f(-3) = 0 to sketch a possible graph of f on the axes provided.

1986 AB 1
Let f be the function defined by $f(x) = 7 - 15x + 9x^2 - x^3$ for all real numbers x.

a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at x = 2.
c. Find the x-coordinates of all points of inflection of f. Justify your answer.

1986 AB 2
Let f be the function given by $f(x) = \dfrac{9x^2 - 36}{x^2 - 9}$.

a. Describe the symmetry of the graph of f.
b. Write an equation for each vertical and each horizontal asymptote of f
c. Find the intervals on which f is increasing.
d. Using the results found in parts (a), (b), and (c), sketch the graph of f.

1986 AB 3 BC 1
A particle moving along the x-axis so that at any time $t \ge 1$ its acceleration is given by $a(t) = \dfrac{1}{t}$. At time t = 1, the velocity of the particle is v(1) = -2 and its position is x(1) = 4.

a. Find the velocity v(t) for $t \ge 1$.
b. Find the position x(t) for $t \ge 1$.
c. What is the position of the particle when it is farthest to the left?

1986 AB 4

Let f be the function defined as follows:

$$f(x)=\begin{cases}|x-1|+2, \text{ for } x<1 \\ ax^2+bx, \text{ for x} \geq 1\text{, where a and b are constants}\end{cases}$$

a. If a = 2 and b = 3, is f continuous of all x? Justify your answer.
b. Describe all values of a and b for which f is a continuous function.
a. For what values of a and b if f both continuous and differentiable?

1986 AB 5 BC 2

Let A(x) be the area of the rectangle inscribed under the curve $y=e^{-2x^2}$ with vertices at (-x, 0) and (x, 0), x $\geq$ 0.

a. Find A(1).
b. What is the greatest value of A(x)? Justify your answer.
c. What is the average value of A(x) on the interval 0 $\leq$ x $\leq$ 2?

1986 AB 6 BC 3

The region enclosed by the graphs of y = $\tan^2$ x, y = $\frac{1}{2}\sec^2$ x, and the y-axis.

a. Find the area of the region R.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region about the x-axis.

1987 AB 1

A particle moves along the x-axis so that its acceleration at any time t is given by a(t) = 6t - 18. At time t = 0 the velocity of the particle is v(0) = 24, and at time t = 1, its position is x(1) = 20.

a. Write an expression for the velocity v(t) of the particle at any time t.
b. For what values of t is the particle at rest?
c. Write an expression for the position x(t) of the particle at any time t.
d. Find the total distance traveled by the particle from t = 1 to t = 3.

1987 AB 2

Let f(x) = $\sqrt{1-\sin x}$.

a. What is the domain of f?
b. Find $f'(x)$.
c. What is the domain of $f'(x)$?
d. Write an equation for the line tangent to the graph of f at x = 0.

1987 AB 3

Let R be the region enclosed by the graphs of $\sqrt[4]{64x}$ and y = x.

a. Find the volume of the solid generated when region R is revolved about the x-axis.

b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable the volume of the solid generated when the region R is revolved about the y-axis.

1987 AB 4

Let f be the function given by f(x) = 2 ln(x^2 + 3) - x with domain -3 $\leq$ x $\leq$ 5.

a. Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

b. Find the x-coordinate of each inflection point of f.

c. Find the absolute maximum value of f(x).

1987 AB 5

A trough is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

a. Find the volume of water in the trough when it is full.

b. What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

c. What is the rate of change in the area of the surface of the water at the instant when the trough is $\frac{1}{4}$ full by volume?

1987 AB 6

Let f be a function such that f(x) < 1 and $f'(x)$ < 0 for all x.

a. Suppose that f(b) = 0 and a < b < c. Write an expression involving integrals for the area of the region enclosed by the graph of f, the lines x = a and x = c, and the x-axis.

b. Determine whether $g(x) = \dfrac{1}{f(x) - 1}$ is increasing or decreasing. Justify your answer.

c. Let he be a differentiable function such that h'(x) < 0 for all x. Determine whether F(x) = H(f(x)) is increasing or decreasing. Justify your answer.

1988 AB 1

Let f be the function given by $f(x)=\sqrt{x^4-16x^2}$.

a. Find the domain of f.
b. Describe the symmetry, if any, of the graph of f.
c. Find $f'(x)$.
d. Find the slope of the line normal to the graph of f at x = 5.

1988 AB 2

A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

a. Find the acceleration a(t) of the particle at any time t.
b. Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.
c. Find the position x(t) of the particle at any time t if x(0) = 0.

1988 AB 3

Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line x = 5.

a. Find the volume of the solid generated by revolving R about the x-axis.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line x = -1.

1988 AB 4

Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

a. Write an equation of the horizontal asymptote for the graph of f.
b. Find the x-coordinate of each critical point of f. For each such x, determine whether f(x) is a relative maximum, a relative minimum, or neither.
c. For what values of x is the graph of f concave down?
d. Using the results found in parts a, b, and c, sketch the graph of y = f(x) in the xy-plane provided.

1988 AB 5

Let R be the region in the first quadrant under the graph of $y=\frac{x}{x^2+2}$ for $0 \leq x \leq \sqrt{6}$.

a. Find the area of R.
b. If the line x = k divides R into two regions of equal area, what is the value of k?
c. What is the average value of $y=\frac{x}{x^2+2}$ on the interval $0 \leq x \leq \sqrt{6}$?

1988 AB 6

Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x) = 18.$

Find f(x). Show your work.